

Linear Programming

Strand: Algebra and Functions

Topic: Linear Programming

Primary SOL: AFDA.5 The student will determine optimal values in problem situations by identifying constraints and using linear programming techniques.

Related SOL: AFDA.2, AFDA.4

Materials

- Maximizing Profit activity sheet (attached)
- Graphing utility
- Graph or grid paper

Vocabulary

constraint, feasible region, intersection, linear, maximized, maximum, minimized, minimum, system of equations, system of inequalities, vertex

Student/Teacher Actions

Time: 90 minutes

1. Using think-pair-share, have students discuss constraints and income as it relates to running a neighborhood lemonade stand. Students should include in their discussions how to determine the maximum amount of lemonade they could produce and other factors that could limit the amount that they could sell in a day.
2. Distribute copies of the Maximizing Profit activity sheet. After students have translated the statements into equations or inequalities, the teacher should verify that the mathematical statements are correct before allowing students to proceed to questions 6-10 on the sheet.

Assessment

- **Questions**
 - What is the feasibility area? Can you have any solutions outside this area? On the perimeter?
 - Will every system of equations have exactly one maximum or one minimum as a solution? Explain why or why not.
- **Journal/writing prompts**
 - Describe another situation where feasibility regions and linear programming may be used in a practical situation.
 - Describe why $x \geq 0$, $y \geq 0$ are normally used in linear programming for constraints, especially for those practical problems that may refer to a business selling a product.
 - Explain to a friend why maximums and minimums only occur at the vertices of the feasibility region.

- **Other Assessments**

- Create an exit slip where students must solve a single system of equations to determine the minimum.
- Have students determine examples where a system of equations might be used to determine a minimum. Students must explain how the constraints would differ from a case looking for the maximum.

Extensions and Connections

- Have students create their own business selling two products and create their own constraints to determine the maximum profit they can earn.
- Students using graphing utilities could solve systems of inequalities by using nonlinear equations. Examples included could be lift equations for airfoils to determine the maximum lift before stall.

Strategies for Differentiation

- Use grid paper to assist students in lining up vertical columns to solve inequalities with one variable isolated.
- Use vocabulary cards for related vocabulary listed above.
- Students with visual impairments could use a talking calculator while graphing the systems of equations during the activity.
- Have students create an organizer with constraints listed on one side and the corresponding equations on the other.

Note: The following pages are intended for classroom use for students as a visual aid to learning.

Maximizing Profit

Many businesses use linear programming to determine how to achieve the maximum profit for a product that they are selling. While the mathematical models they use may be more complicated than those we are using today, the process is very much the same. Assume that you have just been charged with helping to return a struggling furniture business to profitability. The business creates two pieces of furniture: dressers and tables. When graphing, the number of tables should be represented on the y-axis while the number of dressers should be on the x-axis.

Begin by taking the following statements and translating them into equations or inequalities for questions 1-3.

1. Dressers take five hours to build, while tables take three hours to build. The total number of hours must be less than or equal to 400.
2. The total profit is determined by dressers selling for \$325 in profit, while tables sell for \$125 in profit.
3. Dressers cost \$100 and tables cost \$75 for materials manufacture. Total materials cost must not be more than \$9,750.
4. What constraint must we have on the number of dressers sold? Write this as an equation.
5. What constraint must we have on the number of tables sold? Write this as an equation.
6. Solve equations from questions one and five for tables in terms of dressers.
7. On grid paper, graph the equation or inequality in questions three, four, and six, making sure to properly shade when appropriate.

8. Determine the intersection points of the feasibility region.
9. Using the profit equation from question two above, determine the maximum profit based on the coordinates of the feasibility region.
10. Using a graphing utility, determine the profit if the maximum materials cost was increased to \$15,000. You must re-solve for vertices of the feasibility region before determine the profit.